PAPER - I : MODEL PAPER - 07

(JULY 2018)

MATHEMATICS & STATISTICS

COMMERCE

TIME : 1 HR 30 MIN **MARKS : 40**

NOTES :
1. All questions are compulsory
2. Answers to section I and section II must be written in separate ans. Books
3. Graph paper is compulsory for L.P.P.
4. Logarithm table will be provided on demand
5. Figures to the right indicate full marks
6. Answers to every question must be written on new page

Q1. Attempt any six of the following **(12)**

01. if p : It's a day time q : It is warm

Give the verbal statements for the following symbolic statements

a) $p \wedge \sim q$ b) $p \rightarrow q$

SOLUTION

p : It's a day time q : It is warm

a) $p \wedge \sim q$: It is day time and it is not warm

b) $p \rightarrow q$: if it's a day time then it is warm

02. Express the truth of each of the following statements using Venn Diagram

a) No circles are polygons b) Some quadratic equations have equal roots

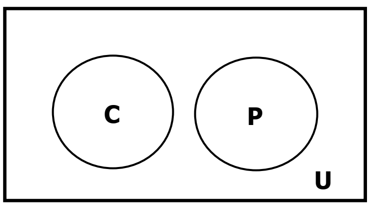
SOLUTION

a) No circles are polygons

C ≡ set of all circles

P ≡ set of all polygons

U ≡ set of all geometrical shapes

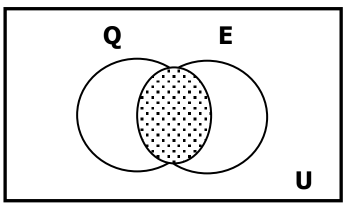


b) Some quadratic equations have equal roots

Q ≡ set of all quadratic equations

E ≡ set of all equations having equal roots

U ≡ set of all equations



03. $2\begin{pmatrix} x & 5 \\ 7 & y-3 \end{pmatrix} + \begin{pmatrix} 3 & -4 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 7 & 6 \\ 15 & 14 \end{pmatrix}$ Find x and y

SOLUTION

$$\begin{pmatrix} 2x & 10 \\ 14 & 2y-6 \end{pmatrix} + \begin{pmatrix} 3 & -4 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 7 & 6 \\ 15 & 14 \end{pmatrix}$$

By Equality of Two Matrices

$$\begin{pmatrix} 2x+3 & 6 \\ 15 & 2y-4 \end{pmatrix} = \begin{pmatrix} 7 & 6 \\ 15 & 14 \end{pmatrix}$$

$2x + 3 = 7$ $\qquad$ $2y - 4 = 14$

$x = 2$ $\qquad$ $y = 9$

04. Find dy/dx if $x = \sin^3\theta$, $y = \cos^3\theta$

SOLUTION

$x = \sin^3\theta$

$$\frac{dx}{d\theta} = 3\sin^2\theta \frac{d}{d\theta}\sin\theta$$

$$= 3\sin^2\theta \,.\cos\theta$$

$y = \cos^3\theta$

$$\frac{dy}{d\theta} = 3\cos^2\theta \frac{d}{d\theta}\cos\theta$$

$$= -3\cos^2\theta \,.\sin\theta$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

$$= \frac{-3\cos^2\theta \,.\sin\theta}{3\sin^2\theta \,.\cos\theta}$$

$$= -\cot\theta$$

05. Find dy/dx if $y = \cos^{-1}\left(2x\sqrt{1-x^2}\right)$

SOLUTION

$y = \cos^{-1} 2x\sqrt{1-x^2}$

Put $x = \sin\theta$

$y = \cos^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$

$y = \cos^{-1}\left(2\sin\theta\sqrt{\cos^2\theta}\right)$

$y = \cos^{-1}(2\sin\theta\cos\theta)$

$y = \cos^{-1}(\sin 2\theta)$

$y = \cos^{-1}\cos(\pi/2 - \theta)$

$y = \pi/2 - \theta$

$y = \pi/2 - \sin^{-1}x$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$$

06. Evaluate $\int x.\log x\, dx$

SOLUTION

$$\log x\int x\,dx - \int\left(\frac{d}{dx}\log x\int x\,dx\right)dx$$

$$\log x\,\frac{x^2}{2} - \int \frac{1}{x}\,\frac{x^2}{2}\,dx$$

$$\frac{x^2}{2}\,.\log x - \frac{1}{2}\int x\,dx$$

$$\frac{x^2}{2}\,.\log x - \frac{x^2}{4} + c$$

07. The cost C of producing x articles is given as $C = x^3 - 16x^2 + 47x$. For what values of x the average cost is decreasing

SOLUTION

$$C_A = \frac{C}{x} = x^2 - 16x + 47$$

For average cost decreasing

$$\frac{dC_A}{dx} < 0$$

$2x - 16 < 0$

$x < 8$

08. $\displaystyle\int_0^1 \frac{1}{1+x^2}\,dx$

$\left[\tan^{-1}x\right]_0^1$

$\tan^{-1}1 - \tan^{-1}0$

$\pi/4$

Q2. (A) Attempt any TWO of the following **(06)**

Q2A

01. Solve the following equations by reduction method

$x + y + z = 6$, $3x - y + 3z = 10$,

$5x + 5y - 4z = 3$

SOLUTION

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & -1 & 3 \\ 5 & 5 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ 3 \end{bmatrix}$$

$R_2 - 3R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -4 & 0 \\ 5 & 5 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ 3 \end{bmatrix}$$

$R_3 - 5R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -4 & 0 \\ 0 & 0 & -9 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ -27 \end{bmatrix}$$

$$\begin{bmatrix} x + y + z \\ -4y \\ -9z \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ -27 \end{bmatrix}$$

By equality of matrices

$-4y = -8 \quad \therefore y = 2$

$-9z = -27 \quad \therefore z = 3$

$x + y + z = 6 \quad \therefore x = 1$

SS { 1,2,3}

02. $\displaystyle\int \frac{2x+1}{(x+1)(x-2)}\,dx$

SOLUTION

$$\frac{2x+1}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2}$$

$2x + 1 = A(x-2) + B(x+1)$

Put $x = 2$

$5 = B(3) \quad \therefore B = 5/3$

Put $x = -1$

$-1 = A(-3) \quad \therefore A = 1/3$

$$\frac{2x+1}{(x+1)(x-2)} = \frac{1/3}{x+1} + \frac{5/3}{x-2}$$

BACK INTO THE SUM

$$= \int \frac{1/3}{x+1} + \frac{5/3}{x-2}\,dx$$

$$= \frac{1}{3}\log|x+1| + \frac{5}{3}\log|x-2| + c$$

03.

$$I = \int_0^1 x\,(1-x)^{3/2}\,dx$$

Q2A

SOLUTION

USING $\int_0^a f(x)dx = \int_0^a f(a-x)\,dx$

WE CHANGE 'x' **TO** '1 – x'

$$I = \int_0^1 (1-x)\,.\,(1-(1-x))^{3/2}\,dx$$

$$I = \int_0^1 (1-x).\,x^{3/2}\,dx$$

$$I = \int_0^1 (x^{3/2} - x^{5/2})\,dx$$

$$I = \left[\frac{x^{5/2}}{\frac{5}{2}} - \frac{x^{7/2}}{\frac{7}{2}}\right]_0^1$$

$$I = \left[\frac{2}{5}x^{5/2} - \frac{2}{7}x^{7/2}\right]_0^1$$

$$I = \frac{2}{5} - \frac{2}{7}$$

$$I = \frac{14-10}{35}$$

$$I = \frac{4}{35}$$

Q2. (B) Attempt any TWO of the following (08)

Q2B

01. Using rules of negations , write the negation of the following

a) $p \wedge (q \rightarrow r)$

SOLUTION

$\sim [p \wedge (q \rightarrow r)]$

$\equiv \sim p \vee \sim(q \rightarrow r)$ .....De Morgan's Law

$\equiv \sim p \vee (q \wedge \sim r)$ ..... $\sim(P \rightarrow Q) \equiv P \wedge \sim Q$

b) $\sim p \vee \sim q$

SOLUTION

$\sim [\sim p \vee \sim q]$

$\equiv \sim(\sim p) \wedge \sim(\sim q)$ .....De Morgan's Law

$\equiv p \wedge q$

02. a manufacturing company produces x items at the total cost of (180 + 4x). The demand function of this product is p = 240 – x . Find x for which the profit is increasing

SOLUTION

$R = px$

$= (240 - x)x$

$= 240x - x^2$

$C = 180 + 4x$

$\pi = R - C$

$= 240x - x^2 - 180 - 4x$

$= 236x - x^2 - 180$

For profit increasing

$$\frac{d\pi}{dx} > 0$$

$236 - 2x > 0$

$236 > 2x$

$x < 118$

Q2B

03. if the function given below is continuous at $x = 2$ and $x = 4$ then find a & b

$$f(x) = \begin{cases} x^2 + ax + b & ; \quad x < 2 \\ 3x + 2 & ; \quad 2 \le x \le 4 \\ 2ax + 5b & ; \quad 4 < x \end{cases}$$

SOLUTION

PART – 1

STEP 1

$$\lim_{x \to 2-} f(x)$$

$$= \lim_{x \to 2} x^2 + ax + b$$

$$= 2^2 + a(2) + b$$

$$= 4 + 2a + b$$

STEP 2

$$\lim_{x \to 2+} f(x)$$

$$= \lim_{x \to 2} 3x + 2$$

$$= 3(2) + 2 = 8$$

STEP 3

$$f(2) = 3(2) + 2 = 8$$

STEP 4

Since the f is continuous at $x = 2$

$$\lim_{x \to 2-} f(x) = \lim_{x \to 2+} f(x) = f(2)$$

$$4 + 2a + b = 8 = 8$$

$$2a + b = 4 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

PART – 2

STEP 1

$$\lim_{x \to 4-} f(x)$$

$$= \lim_{x \to 4} 3x + 2$$

$$= 3(4) + 2 = 14$$

STEP 2

$$\lim_{x \to 4+} f(x)$$

$$= \lim_{x \to 4} 2ax + 5b$$

$$= 2a(4) + 5b$$

$$= 8a + 5b$$

STEP 3

$$f(4) = 3(4) + 2 = 14$$

STEP 4

Since the f is continuous at $x = 4$

$$\lim_{x \to 4-} f(x) = \lim_{x \to 4+} f(x) = f(4)$$

$$14 = 8a + 5b = 14$$

$$8a + 5b = 14 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Solving (1) and (2) : $a = 3$, $b = -2$

Q3.

(A) Attempt any TWO of the following (06)

01. if $A = \begin{bmatrix} 7 & 1 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$;

Verify : $|AB| = |A|.|B|$

SOLUTION

LHS

AB

$$= \begin{bmatrix} 7 & 1 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 7+3 & 14-1 \\ 2+15 & 4-5 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 13 \\ 17 & -1 \end{bmatrix}$$

$$|AB| = 10(-1) - 17(13) = -10 - 221$$
$$= -231$$

RHS

$$= |A| . |B|$$

$$= \begin{vmatrix} 7 & 1 \\ 2 & 5 \end{vmatrix}\begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix}$$

$$= (35 - 2) . (-1 - 6)$$

$$= 33 (-7)$$

02. Q3A

$$\int \frac{1}{x\left[(\log x)^2 + 4\right]} dx$$

PUT $\log x = t$

$$\frac{1}{x} . dx = dt$$

THE SUM IS

$$= \int \frac{1}{t^2 + 4} dt$$

$$= \int \frac{1}{t^2 + 2^2} dt$$

$$= \frac{1}{a} \tan^{-1} \frac{t}{a} + c$$

$$= \frac{1}{2} \tan^{-1} \frac{t}{2} + c$$

Resubs.

$$= \frac{1}{2} \tan^{-1}\left(\frac{\log x}{2}\right) + c$$

03. Find the volume of the solid generated by rotating the area bounded by $x^2 + y^2 = 36$ and the lines $x = 0$, $x = 3$ about the x – axis

SOLUTION

$$V = \pi \int_0^3 y^2 . dx$$

$$= \pi \int_0^3 36 - x^2 . dx$$

$$= \pi \left[36x - \frac{x^3}{3}\right]_0^3$$

$$= \pi \left\{\left(108 - \frac{27}{3}\right) - \left(0 - \frac{0}{3}\right)\right\}$$

$$= \pi (108 - 9)$$

$$= 99\pi \text{ cubic units}$$

(B)

Attempt any TWO of the following (08)

Q3B

01.

if f is continuous at x = 0 , then find f(0) where

$$f(x) = \frac{(3^{\sin x} - 1)^2}{x \cdot \log(1 + x)} \quad ; \quad x \neq 0$$

SOLUTION

$$\lim_{x \to 0} f(x)$$

$$\lim_{x \to 0} \frac{(3^{\sin x} - 1)^2}{x \cdot \log(1 + x)}$$

Divide N and D by $\sin^2 x$, $\sin^2 x \neq 0$

$$\lim_{x \to 0} \frac{\frac{(3^{\sin x} - 1)^2}{\sin^2 x} \sin^2 x}{x \cdot \log(1 + x)}$$

Divide N and D by x^2 , $x^2 \neq 0$

$$\lim_{x \to 0} \frac{\frac{(3^{\sin x} - 1)^2}{\sin^2 x} \frac{\sin^2 x}{x^2}}{\frac{x \cdot \log(1 + x)}{x^2}}$$

$$\lim_{x \to 0} \frac{\left(\frac{3^{\sin x} - 1}{\sin x}\right)^2 \left(\frac{\sin x}{x}\right)^2}{\frac{\log(1 + x)}{x}}$$

$$= \frac{(\log 3)^2 \cdot (1)^2}{1}$$

$$= (\log 3)^2$$

Since f(x) is continuous at x = 0

$$f(0) = \lim_{x \to 0} f(x)$$

$$= (\log 3)^2$$

02.

the processing cost of x bags is $\frac{2x^3}{3} - 48x^2$,

and packing & dispatching cost is (1289x + 3750)

Find the number of bags to be manufactured so as to minimize the marginal cost . Also find the marginal cost for that number of bags

SOLUTION

$$C = \frac{2x^3}{3} - 48x^2 + 1289x + 3750$$

$$C_M = \frac{dC}{dx}$$

$$= \frac{6x^2}{3} - 96x + 1289$$

$$= 2x^2 - 96x + 1289$$

$$\frac{dC_M}{dx} = 4x - 96$$

$$\frac{d^2C_M}{dx^2} = 4$$

$$\frac{dC_M}{dx} = 0$$

$4x - 96 = 0 \qquad x = 24$

$$\left.\frac{d^2C_M}{dx^2}\right|_{x = 24} = 4 > 0$$

C_M is minimum at x = 24

$$\left.C_M\right|_{x = 24} = 2(24)2 - 96(24) + 1289$$

$$= 2(576) - 2304 + 1289$$

$$= 1152 - 2304 + 1289$$

$$= 137$$

03. $x = \dfrac{4t}{1+t^2}$; $y = 3\dfrac{1-t^2}{1+t^2}$

Show that $\dfrac{dy}{dx} = -\dfrac{9x}{4y}$

$$\mathbf{x = \frac{4t}{1+t^2}}$$

$$\mathbf{x = 2\frac{2t}{1+t^2}}$$

Put $t = \tan\theta$

$$x = 2\frac{2\tan\theta}{1+\tan^2\theta}$$

$$x = 2\sin 2\theta \qquad \ldots\ldots\ldots\ (1)$$

$$\frac{dx}{d\theta} = 2\cos 2\theta \cdot \frac{d\,2\theta}{d\theta}$$

$$= 2\cos 2\theta \cdot 2 \qquad = 4\cos 2\theta$$

$$\mathbf{y = 3\frac{1-t^2}{1+t^2}}$$

Put $t = \tan\theta$

$$y = 3\frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$y = 3\cos 2\theta \qquad \ldots\ldots\ldots\ (2)$$

$$\frac{dy}{d\theta} = 3. -\sin 2\theta \cdot \frac{d\,2\theta}{d\theta}$$

$$= -3\sin 2\theta \cdot 2 \qquad = -6\sin 2\theta$$

Now

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$\frac{dy}{dx} = \frac{-6\sin 2\theta}{4\cos 2\theta}$$

$$\frac{dy}{dx} = -\frac{3\sin 2\theta}{2\cos 2\theta}$$

$$\frac{dy}{dx} = -\frac{3\frac{x}{2}}{2\frac{y}{3}} \qquad \text{FROM (1) \& (2)}$$

$$\frac{dy}{dx} = -\frac{9x}{4y}$$

DO NOT STOP

GET READY FOR NEXT